Flavour-Dependent CP Violation and Natural Suppression of the Electric Dipole Moments

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We revisit the supersymmetric CP problem and find that it can be naturally resolved if the origin of CP violation is closely related to the origin of flavour structures. In this case, the supersymmetry breaking dynamics do not bring in any *new* CP-violating phases. This mechanism requires hermitian Yukawa matrices which naturally arise in models with a U(3) flavour symmetry. The neutron electric dipole moment (NEDM) is predicted to be within one-two orders of magnitude below the current experimental limit. The model also predicts a strong correlation between $A_{CP}(b \to s\gamma)$ and the NEDM. The strong CP problem is mitigated although not completely solved.

The tight experimental bounds on the EDMs are an important tool for understanding supersymmetry breaking. For example, they might indicate that the SUSY parameters happen to fall in small regions of the parameter space where there are accidental cancellations between various contributions or where the supersymmetric phases are small [1]. Another possibility is that the sfermions of the first two generations have masses in the TeV range. Alternatively, and perhaps more naturally, these bounds might indicate that the *flavour-independent* SUSY phases are forced to vanish by the underlying theory. If this is correct, it seems likely that CP violation is intimately related to the *origin* of the flavour structures in the model rather than the origin of supersymmetry breaking. In other words the source of CP violation is associated with whatever generates the SUSY Yukawa interactions, and the supersymmetry breaking dynamics do not contribute any further CP violation. Under this assumption it may be possible to redefine the fields such that the moduli and dilaton auxiliary fields have *real* vacuum expectation values (VEVs). Such behaviour has been observed in some explicit string models in which CP violation is derived from first principles in effective type I string models [2].

Since the CP breaking scale is related to the scale at which the flavor symmetry gets broken, it must be far above the SUSY breaking scale, probably close to the string or GUT scales. If the underlying structure indeed allows only quantities with a non-trivial flavor structure to have CP-violation, this automatically implies that the gaugino masses and the μ term are *real* (we illustrate it below). On the other hand, the trilinear soft couplings may still involve CP-violating phases, and $\mathcal{O}(1)$ phases of the diagonal elements of the A–terms induce unacceptably large EDMs by themselves. Following the above logic, we conclude that they too are prohibited by the mechanism generating CP violation. An important point to note however, is that this is generally a *basis dependent* requirement, which is to say that the diagonal A–terms must be real in the quark mass basis. Conversely, even if the A–terms are all real to begin with, diagonalizing the quark Yukawas generally causes phases to appear in the diagonal terms.

This fact strongly suggests that the flavour structure forces the diagonal A–terms to be real in *any* basis. In what follows we consider the only possibility of which we are aware, that the flavour structure is hermitian. Hermitian Yukawa matrices naturally occur in models with a horizontal flavour symmetry [3] and left-right symmetric models [4]. As we shall shortly see, if SUSY and CP breaking are decoupled, this guarantees a nearly hermitian structure for the A–terms and hence suppressssed EDMs. This setting appears in string models with hidden sector fields in the *adjoint* representation of certain symmetry groups.

Models with non-universal A-terms have recently attracted considerable attention [5]. In these models, in order to avoid a conflict with the EDM bounds, the condition of small diagonal phases (or EDM cancellations) is normally imposed by hand without referring to a particular mechanism. In contrast, the EDM in models with hermitian flavour structures is a derived quantity. We find that the neutron EDM is predicted to be within one or two orders of magnitude below the current experimental limit. Also, we show that the SUSY CP-phases in this class of models can have a significant effect on the B and K physics observables, in particular the $b \to s\gamma$ CP-asymmetry and the $K^0 - \bar{K}^0$ mixing. An interesting feature of this scenario is a strong correlation between $A_{CP}(b \to s\gamma)$ and the neutron EDM, i.e. a large ($\sim 10\%$) CP-asymmetry in the $b \to s\gamma$ decay implies that the NEDM is of order $10^{-26} e \cdot cm$ which is just below the current limit. This prediction can be tested in the near future.

We will study the implications of hermitian Yukawa matrices in the supergravity framework:

$$Y^{u\dagger} = Y^u \ , \ Y^{d\dagger} = Y^d \ , \ Y^{l\dagger} = Y^l \ ,$$

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$$\text{Arg}(\mu) = \text{Arg}(M_i) = 0 \ . \qquad (1)$$

In supergravity models, the soft SUSY breaking parameters are given in terms of the Kähler potential and the superpotential. In particular, the trilinear parameters are written as [6]

$$A_{\alpha\beta\gamma} = F^m \big[\hat{K}_m + \partial_m \log Y_{\alpha\beta\gamma} - \partial_m \log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \big] \ . \quad (2)$$

Here the Latin indices refer to the hidden sector fields while the Greek indices refer to the observable fields; the Kähler potential is expanded in observable fields as $K = \hat{K} + \tilde{K}_\alpha |C^\alpha|^2 + ...$ and $\hat{K}_m \equiv \partial_m \hat{K}$.

Note that $\tilde{K}_\alpha$ is always real; $\hat{K}_m$ is real if $\hat{K}$ is a function of $h_m + h_m^*$. The F-terms are real according to the basic assumption of the model that the SUSY breaking dynamics do not violate CP. For definiteness, let us fix the index α to refer to the Higgs fields, and indices β and γ to refer to the left-handed and right-handed fields, respectively. The resulting A–terms are hermitian in the generational indices if the derivatives of the Kähler potential $\tilde{K}_{\beta,\gamma}$ are either generation-independent (for the left and right fields separately) or the same for the left and right fields of the same generation ($F^m \hat{K}_m + F^m \partial_m \log Y_{\alpha\beta\gamma}$ is hermitian). These conditions are satisfied in simple Type I and heterotic string models. Consequently, the quantities $\hat{A}_{\alpha\beta\gamma} \equiv A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma}$ are hermitian with respect to the generational indices; i.e.

$$\hat{A}^{u\dagger} = \hat{A}^u \ , \ \hat{A}^{d\dagger} = \hat{A}^d \ , \ \hat{A}^{l\dagger} = \hat{A}^l \ . \qquad (3)$$

The flavour structure of the $\hat{A}$'s can be quite different from that of the Y's. Note that even if the Yukawa couplings have a negligible modular dependence and $A_{\alpha\beta\gamma}$ are real (and symmetric), the quantities $\hat{A}$ responsible for the phenomenology must involve complex phases. The other soft breaking parameters are real as they are independent of $Y_{\alpha\beta\gamma}$. In what follows, we will assume for simplicity a universal soft scalar mass parameter*.

All these quantities are subject to renormalization group (RG) running which may not preserve the hermiticity. For instance, the RG evolution of Y^u involves a term $Y^d Y^{d\dagger} Y^u$ which is not hermitian; the amount of generated "non-hermiticity" depends on $[Y^u, Y^d]$ and therefore is suppressed by the off-diagonal entries of the Yukawas. Generally, the Yukawa running effects are negligible for the first two generations anyway, so potentially non-negligible non-hermitian contribution can come only from the third generation. Similar considerations apply

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*This is the case if the only non-trivial flavour structure in the model comes from the Yukawa matrices or when the derivatives of the Kähler potential are small compared to $m_{3/2}^2$.
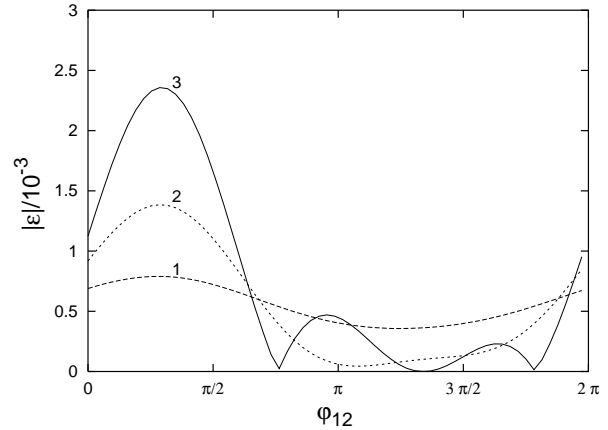

FIG. 1. SUSY contribution to $|\varepsilon|$ as a function of the phase ϕ_{12} for $m_0 \simeq 150$ GeV, $\tan\beta = 3$, and $m_{\chi^+} \simeq 100$ GeV. Curve 1: $|A_{12}| = m_0$; curve 2: $|A_{12}| = 3m_0$; curve 3: $|A_{12}| = 5m_0$. The observed value is $|\varepsilon| \simeq 2.26 \times 10^{-3}$.

to the RG evolution of the A–terms. These radiative effects induce non-zero EDMs as discussed below. The phases of the gaugino masses and the μ-term are RG-invariant and remain zero.

Therefore, the Yukawas and A-terms stay hermitian to a good degree even at low energies. The hermitian Yukawas are diagonalized by a unitary transformation,

$$u \to V^u u \ , \ Y^u \to V^{u\dagger} Y^u V^u \qquad (4)$$

and similarly for the down quark and lepton fields. The resulting CKM matrix is given by $V_{CKM} = V^{u\dagger} V^d$. If we transform the sfermion fields in the same way as we transform the fermion fields, we will go over to a basis known as the "super-CKM" (SCKM) basis in the literature. The A–terms transform accordingly;

$$\hat{A}^u \to V^{u\dagger} \hat{A}^u V^u \ , \ \hat{A}^d \to V^{d\dagger} \hat{A}^d V^d \ . \qquad (5)$$

The A–terms in this super-CKM basis remain hermitian and therefore have real diagonal elements. As a result, the flavour-conserving mass insertions appearing in the EDM calculations $(\delta_{ii}^{d(u)})_{LR} = \frac{1}{\tilde{m}^2}((\hat{A}_{SCKM}^{d(u)\dagger})_{ii} v_{1(2)} - Y_i^{d(u)} \mu v_{2(1)})$ (no sum over i) are real to a good degree. This provides a natural suppression of the EDMs. Note that hermiticity of the Yukawas is crucial for this suppression. If we give it up, A–terms which are hermitian or even real at high energies will gain diagonal phases due to the transformation of the type (5) with $V_L \neq V_R$.

Our numerical studies show that the radiative effects producing non-hermitian pieces in the Yukawas and A–terms typically lead to a neutron EDM between 10^{-27} and $10^{-28} e \cdot cm$ and an electron EDM of about $10^{-33} e \cdot cm$. In some cases, when the condition of a large CP-asymmetry in $b \to s\gamma$ is imposed, the NEDM can be as large as $\mathcal{O}(10^{-26}) e \cdot cm$. Generically the model predicts

the NEDM to be within two orders of magnitude below the current experimental bound.

It is interesting to note that the strong CP problem is mitigated in this model. The $\bar{\theta}$ parameter vanishes both at the tree and the 1-loop leading log levels. This can be seen from the fact that $\frac{d}{dt}\mathrm{Det}Y = \mathrm{Det}Y\ \mathrm{Tr}\left[Y^{-1}\frac{d}{dt}Y\right]$ with $\mathrm{Tr}\left[Y^{-1}\frac{d}{dt}Y\right]$ being real (see the RGEs in [7]). The gluino phase is RG-invariant and vanishes at the high energy scale. As a result, the $\bar{\theta}$ parameter remains zero under the RG flow. However, there are one loop finite contributions which generate both a gluino phase and $\mathrm{Arg}(\mathrm{Det}Y^{u,d})$. The strong CP problem in our model is milder than it normally is in SUSY models, yet it still exists.

The complex phases appearing in the off-diagonal elements of the A–terms can have a significant impact on the kaon and B physics. In our numerical studies we used the following representative GUT scale hermitian Yukawa matrices

$$Y^u = \begin{pmatrix} 4.1\times 10^{-4} & 6.9\times 10^{-4}\,\mathrm{i} & -1.4\times 10^{-2} \\ -6.9\times 10^{-4}\,\mathrm{i} & 3.5\times 10^{-3} & -1.4\times 10^{-5}\,\mathrm{i} \\ -1.4\times 10^{-2} & 1.4\times 10^{-5}\,\mathrm{i} & 6.9\times 10^{-1} \end{pmatrix},\ (6)$$

$$Y^d = \begin{pmatrix} 1.3\times 10^{-4} & (2.0+1.8\,\mathrm{i})\times 10^{-4} & -4.4\times 10^{-4} \\ (2.0-1.8\,\mathrm{i})\times 10^{-4} & 9.3\times 10^{-4} & 7.0\times 10^{-4}\,\mathrm{i} \\ -4.4\times 10^{-4} & -7.0\times 10^{-4}\,\mathrm{i} & 1.9\times 10^{-2} \end{pmatrix},$$

which at low energies reproduce the quark masses and the CKM matrix (we use $\tan\beta = 3$ throughout the paper). In this analysis it is necessary to use the complete set of the MSSM RG equations as given in Ref. [7].

We find that the SUSY contribution to the ε' parameter is negligible. This occurs due to severe cancellations between the contributions involving $(\delta_{12}^d)_{LR}$ and $(\delta_{12}^d)_{RL}$ mass insertions (we use the standard definitions of [8]). Due to the hermiticity $(\delta_{12}^d)_{LR} \simeq (\delta_{12}^d)_{RL}$, whereas they contribute to ε' with opposite signs.

On the other hand, the SUSY contribution to the ε parameter can be substantial. The gluino–down squark contribution to ε does not suffer from similar cancellations. In Fig. 1 we plot the values of $|\varepsilon|$ versus the phase (ϕ_{12}) of the off–diagonal element A_{12} for three representative values of $|A_{12}|$, namely $|A_{12}| = m_0$ (curve 1), $|A_{12}| = 3m_0$ (curve 2), and $|A_{12}| = 5m_0$ (curve 3). For simplicity we assume $A^d = A^u = A^l$. We fix the universal scalar mass m_0 to be 150 GeV and adjust $m_{1/2}$ such that the lightest chargino mass is around 100 GeV. We have also fixed the other elements of the A-terms to be m_0.

The major contribution to ε comes from the gluino–down squark diagrams with the left-right and right-left mass insertions which in this case interfere constructively. The left-left and right-right mass insertions appear only due to the RG running effects. Their typical values are $\mathcal{O}(10^{-5})$ and $\mathcal{O}(10^{-6})$, respectively, which are too small to produce any significant effect. The value of $(\delta_{12}^d)_{LR}$

which saturates the observed $|\varepsilon| \simeq 2.26 \times 10^{-3}$ is given by $\sqrt{|\mathrm{Im}(\delta_{12}^{\mathrm{d}})_{\mathrm{LR}}^2|} \simeq 3.5 \times 10^{-4}$ for the gluino and squark masses of 500 GeV [8]. Fig.1 demonstrates that increasing the GUT value of $|A_{12}|$ increases the SUSY contribution to ε, as expected. Note that curve 1 never crosses the horizontal axis meaning that the SUSY contribution to ε in this case does not vanish for any value of ϕ_{12}. This occurs due to the non-trivial phases in the Yukawa matrices. The main message of these results is that the SUSY contribution can accommodate large values of ε without violating the EDM constraints. Indeed, we found that the NEDM is about $2 \times 10^{-27} e \cdot cm$ in all of these cases. As we discuss below, these results have implications for the B system as they affect the prediction for the $B^0 - \bar{B}^0$ mixing phase.

In the B system, the observables of primary interest are the CP-asymmetry in the $b \to s\gamma$ decay and the $B^0 - \bar{B}^0$ mixing. In Fig.2 we present a plot showing the dependence of $A_{CP}(b \to s\gamma)$ on the phase ϕ_{23} of the element A_{23} for three values of $|A_{23}| = |A_{33}|$: $|A_{23}| = m_0$ (curve 1), $|A_{23}| = 3m_0$ (curve 2), and $|A_{23}| = 5m_0$ (curve 3). Other elements are taken to be m_0. We automatically impose the condition of the correct branching ratio, so the points not satisfying this constraint are not shown. For values of $|A_{23}|$ close to m_0, the CP-asymmetry is a few percent which is already significantly larger than the SM expectation. As emphasised in Ref. [9], the CP–asymmetry is very sensitive to the magnitude of the elements A_{23} and A_{33}. For a sufficiently large $|A_{23}| = |A_{33}|$ ($\geq 3m_0$), the CP-asymmetry can be as large as 6-8 percent. It could be even larger in other regions of the parameter space.

The magnitudes of A_{23} and A_{33} have a significant impact on the RG evolution of the A-terms. In particular, they are responsible for generating complex phases in $(\delta_{11}^{d(u)})_{LR}$ which produce the NEDM. We found that curve 3 corresponds to the NEDM of $4.1 \times 10^{-26} e \cdot cm$, while for curves 2 and 1 it is $2.6 \times 10^{-26} e \cdot cm$ and $2.5 \times 10^{-27} e \cdot cm$, respectively. This provides an interesting signature of the model: a large CP-asymmetry in the $b \to s\gamma$ decay can be accommodated only if the NEDM is just below the current limit.

The SUSY contribution to the $B^0 - \bar{B}^0$ mixing is small for typical values of the A-terms (i.e. $|(\delta_{13}^d)_{LR}| \lesssim 5 \times 10^{-4}$). Nevertheless, supersymmetry can affect it indirectly by changing the predicted value for $\sin 2\beta$ [10]. As we have shown, SUSY may be responsible for the observed value of ε. If it is, there is no constraint from ε on the CKM matrix. In this case lower values for $\sin 2\beta$ are allowed [11], in better agreement with the recent BaBar and Belle results [12].

Now we turn to the discussion of how hermitian Yukawa matrices can be implemented. One of the possibilities is based on a concept of so-called "real CP-violation" introduced by Masiero and Yanagida [3], and
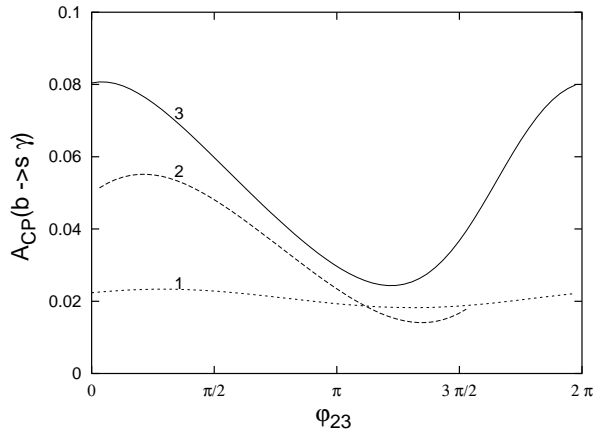
FIG. 2. CP-asymmetry $A_{CP}(b \to s\gamma)$ as a function of the phase ϕ_{23} for $m_0 \simeq 150$ GeV, $\tan\beta = 3$, $m_{\chi^+} \simeq 100$ GeV, and $|A_{23}| = |A_{33}|$. Curve 1: $|A_{23}| = m_0$; curve 2: $|A_{23}| = 3m_0$; curve 3: $|A_{23}| = 5m_0$.

Lavoura [13]. Hermitian Yukawa matrices may appear due to a (gauged) horizontal symmetry $U(3)_H$ which gets broken spontaneously by the VEVs of the real adjoint fields T_α^a ($a = 1..9$; $\alpha = u, d, l, ..$). Some of these *real* VEV's also break CP since some of the components of T_α^a are CP-odd. As a result, CP violation appears in the superpotential through complex Yukawa couplings, whereas the μ-term is real since it arises from a $U(3)_H$ invariant combination of the T's. An effective $U(3)_H$-invariant superpotential of the type $\hat{W} = \frac{g_H}{M} \hat{H}_1 \hat{Q}_i (T_d^a \lambda^a)_{ij} \hat{D}_j$ produces the Yukawa matrix $(Y^d)_{ij} = \frac{g_H}{M} \langle T_d^a \rangle (\lambda^a)_{ij}$, where λ^0 is proportional to the unit matrix and λ^{1-8} are the Gell-Mann matrices.

In a somewhat different context, hermitian Yukawa matrices and real flavour-independent quantities arise in the left-right symmetric models [4]. The left-right symmetry may be too restrictive to satisfy all of the phenomenological constraints even if one allows for a significant fine-tuning (see the last reference in [4]). The B-factories' data will be necessary to make a conclusive statement concerning the viability of such models.

In view of the current high level of activity in the subject, it is of interest to see whether and how the fields needed may arise in string theory. Real adjoint fields may only arise from non-supersymmetric states, while complex adjoints could be part of a chiral multiplet (in which case hermitian Yukawas appear only if all the VEVs are real). Massless, adjoint, chiral multiplets occur in (weakly coupled) heterotic string models when the affine lie algebra is realized at level $k \geq 2$. (See, for example, [14].) Such models enable the construction of ($k = 2$) string GUTs based on $SU(5)$ or $SO(10)$, although realistic models are difficult to obtain [15], [16]. However, in none of the models is there more than one such adjoint multiplet, and their construction is such that the scalars arise as continuous moduli, having no self coupling, and

whose VEVs must therefore be fixed by non-perturbative effects. Adjoint matter representations also arise in Type IIB orientifold models with discrete torsion [17], both as a chiral field in a supersymmetric sector (of the $Z_2 \times Z_4$ model with $U(8)$ gauge group), and as a scalar field in a non-supersymmetric sector (of the $Z_2 \times Z_6$ model with $USp(8) \times U(4)$ gauge group). Either of these can, in principle, accommodate the standard model gauge group together with a $U(3)$ horizontal symmetry, but as before in neither is there more than one copy of the adjoint. Currently, we are not aware of string models in which all of the required features arise. This subject requires further study.

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